

Cape Florida Lighthouse: Lore and Calculations

Activity 2 - Meade Extension

Given: In 1852, George G. Meade, later the Union General at Gettysburg, designed the plan to elevate the Cape Florida Lighthouse tower to its present height of 95 feet by building atop the original structure. The bottom of the extension matches the top of the original tower, and the outer diameter of the top of the extension is 10' 6" and the inner diameter is 7' 6", as indicated on the original blueprints located in both the National Archives and the State Archives of Florida.

Tasks:

1. Either modify your previous scale drawing or use graph paper and a ruler to create a new scale drawing to represent the lighthouse tower that meets the extension criteria.
2. Determine if the Meade extension maintained the silhouette (profile) of the lighthouse.
3. Calculate the percent of increase in the lighthouse tower's height.
4. Calculate the percent of increase in the lighthouse tower's volume of brick and mortar.
5. Assume bricks used for the tower extension had the same dimensions as the original bricks, $\frac{3}{4}$ " \times $3\frac{3}{8}$ " \times 2" with $\frac{3}{8}$ " mortar joints. Determine the number of bricks used for the extension.
6. Park Services Specialists at Bill Baggs Cape Florida State Park, where the Cape Florida Lighthouse is located, say one of the most frequently asked questions is, "How many bricks are in the lighthouse?" Answer this question.
7. Over a century of neglect, the lighthouse tower was windswept, wave-washed, and sandblasted. During the 1992-1996 restoration, more than 30,000 bricks were recast in Tennessee to match the density of the original bricks. What percent of the bricks in the lighthouse tower are new?
8. Determine the lateral surface area, in square feet, of the total exterior of the extended lighthouse tower.
9. Suppose a gallon of paint covers approximately 350 square feet. Calculate the number of gallons needed to cover the total exterior of the extended lighthouse tower with one coat of paint.

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Tasks:

1. Either modify your previous scale drawing or use graph paper and a ruler to create a new scale drawing to represent the lighthouse tower that meets the extension criteria. See last page
2. Determine if the Meade extension maintained the silhouette (profile) of the lighthouse.
No, there is a change of slope. The slant height of the extension is steeper than that of the original tower.
3. Calculate the percent of increase in the lighthouse tower's height. (95 - 65)/65 = 0.461 ≈ **46%**
4. Calculate the percent of increase in the lighthouse tower's volume of brick and mortar.

Volume of lighthouse tower extension = $V_{\text{outer frustum}} - V_{\text{inner frustum}}$

Vol. of outer frustum = $\frac{1}{3}\pi h(r_1^2 + r_1r_2 + r_2^2)$

$V = \frac{1}{3}\pi(30)(6^2 + (6)(5.25) + 5.25^2)$

$V = \pi(10)(95.0625)$

$V = 950.625\pi$

$V \approx 2,986.476$ cubic feet

Vol. of inner frustum = $\frac{1}{3}\pi h(r_1^2 + r_1r_2 + r_2^2)$

$V = \frac{1}{3}\pi(30)(4^2 + (4)(3.75) + 3.75^2)$

$V = \pi(10)(45.0625)$

$V = 450.625\pi$

$V \approx 1,415.680$ cubic feet

Volume of lighthouse tower extension $\approx 2,986.476 - 1,415.680 \approx$ **1,570.796 cubic feet**

1,570.796 cubic feet / 9,529.497 cubic feet = 0.164 = **16.4% increase**

5. Assume bricks used for the tower extension had the same dimensions as the original bricks,

$\frac{3}{4}$ " \times $3\frac{3}{8}$ " \times 2" with $\frac{3}{8}$ " mortar joints. Determine the number of bricks used for the extension.

$$1,570.796 \text{ cubic feet} \times 1,728 \text{ cubic inches per cubic foot} = 2,714,335.488 \text{ cubic inches}$$

$$2,714,335.488 \div 72.363 \text{ cubic inches per brick (as determined in Activity 1)} = 37,509.991 \approx$$

37,510 bricks

6. Park Services Specialists at Bill Baggs Cape Florida State Park, where the Cape Florida Lighthouse is located, say one of the most frequently asked questions is, "How many bricks are in the lighthouse?" Answer this question.

$$227,561 \text{ bricks in original tower} + 37,510 \text{ bricks in extension} = \mathbf{265,071 \text{ bricks in total}}$$

7. Over a century of neglect, the lighthouse tower was windswept, wave-washed, and sandblasted. During the 1992-1996 restoration, more than 30,000 bricks were recast in Tennessee to match the density of the original bricks. What percent of the bricks in the lighthouse tower are new?

$$30,000 \div 265,071 = 0.113 = \mathbf{11.3\%}$$

8. Determine the lateral surface area, in square feet, of the total exterior of the extended lighthouse tower.

$$\text{Lateral Surface Area of Extension} = F = \pi(r_1 + r_2)\sqrt{h^2 + (r_1 - r_2)^2}$$

$$F = \pi(6 + 5.25)\sqrt{30^2 + (6 - 5.25)^2}$$

$$F = \pi(11.25)\sqrt{900.5625}$$

$$F \approx 337.605\pi \text{ square feet} \approx 1,060.618 \text{ square feet}$$

$$\text{Lateral Surface Area of Original Tower} \approx 3,691.289 \text{ square feet}$$

$$\text{Total Lateral Surface Area} \approx 1,060.618 \text{ square feet} + 3,691.289 \text{ square feet} \approx \mathbf{4,751.907 \text{ sq. feet}}$$

9. Suppose a gallon of paint covers approximately 350 square feet. Calculate the number of gallons needed to cover the total exterior of the extended lighthouse tower with one coat of paint.

$$4,751.907 \div 350 = 13.57 \approx \mathbf{14 \text{ gallons}}$$

Profile of Cape Florida Lighthouse Tower with Meade Extension

